AP Physics C Free Response Practice – Work and Energy – ANSWERS

1973M1

a. Free-Body Diagrams
- free-body diagram for m_1 contains
  - i. friction force F_f directed to the left
  - ii. pulling force F directed to the right
  - iii. gravitational force m_1g directed downwards
  - iv. normal force of 2 on 1 $F_{N(2\text{ on }1)}$ directed upwards
- free-body diagram for m_2 contains
  - i. friction force F_f directed to the right
  - ii. gravitational force m_2g directed downwards
  - iii. normal force of 1 on 2 $F_{N(1\text{ on }2)}$ directed downwards
  - iv. normal force of surface on 2 $F_{N(S\text{ on }2)}$ directed upwards

b. Using Newton's Second Law and the free-body diagrams we get:
$\Sigma F = m_1a_1$; $F - F_f = m_1a_1$; $F - \mu m_1g = m_1a_1$
$\Sigma F = m_2a_2$; $F_f = m_2a_2$; $\mu m_1g = m_2a_2$
$a_2 = \mu(m_1/m_2)g$ and $a_1 = (F/m_1) - \mu g$

c. Applying kinematics to m_1: $x_1 = v_it + \frac{1}{2}(a_{rel})t^2$, where $x = l$, $v_i = 0$ and $a_{rel} = a_1 - a_2$: the acceleration of m_1 relative to m_2. Solving for t gives:

$$t = \sqrt{\frac{2l}{\left(\frac{F}{m_1}\right) - \mu g\left(1 + \frac{m_1}{m_2}\right)}}$$

d. $\Delta E = W_f = -F_fd = -\mu m_1gl$

1981M2

a. $(2M)gh = \frac{1}{2}(2M)v^2$

$h = L/2$ so $v = (gL)^{1/2}$ at the bottom

During the upswing, $\frac{1}{2} M v_{swing}^2 = MgH$ where $H = L(1 - \sqrt{2}/2)$

$$v_s = \sqrt{gL}\left(\sqrt{2-\sqrt{2}}\right)$$

During the jump momentum is conserved

$2Mv = Mv_c + Mv_s$

$$v_c = \sqrt{gL}\left(2-\sqrt{2-\sqrt{2}}\right)$$

<u>1982M1</u>

a. Apply energy conservation, set the top of the spring as h=0, therefore H at start = Lsin θ = 6 sin30 = 3 m

   $U_{top} = K_{bot}$ $mgh = ½ mv^2$ $(9.8)(3) = ½ (v^2)$ $v = 7.67$ m/s

b. Set a new position for h=0 at the bottom of the spring. Apply energy conservation comparing the h=0 position and the initial height location. Note: The initial height of the box will include both the y component of the initial distance along the inclined plane plus the y component of the compression distance Δx.
   $h = L \sin\theta + \Delta x \sin\theta$

   $U_{top} = U_{sp}(bot)$
   $mgh = ½ k \Delta x^2$
   $mg(L \sin\theta + \Delta x \sin\theta) = ½ k \Delta x^2$
   $(20)(9.8)(6 \sin 30 + 3 \sin 30) = ½ k (3)^2$ $k = 196$ N/m

c. The speed is NOT a maximum when the block first hits the spring. Although the spring starts to push upwards against the motion of the block, the upwards spring force is initially less than the x component of the weight pushing down the incline (F_{gx}) so there is still a net force down the incline which makes the box accelerate and gain speed. This net force will decrease as the box moves down and the spring force increases. The maximum speed of the block will occur when the upwards spring force is equal in magnitude to the force down the incline such that Fnet is zero and the box stops accelerating down the incline. Past this point, the spring force becomes greater and there is a net force acting up the incline which slows the box until it eventually and momentarily comes to rest in the specified location.

C1985M2

a. We use $F_{net} = 0$ for the initial brink of slipping point. $F_{gx} - f_k = 0$ $mg \sin\theta = \mu_s(F_n)$
$mg \sin\theta = \mu_s\, mg \cos\theta$ $\mu_s = \tan\theta$

b. Note: we cannot use the friction force from part a since this is the static friction force, we would need kinetic friction. So instead we must apply W_{nc} = energy loss = $\Delta K + \Delta U + \Delta U_{sp}$. ΔK is zero since the box starts and ends at rest, but there is a loss of gravitational U and a gain of spring U so those two terms will determine the loss of energy, setting final position as h=0. Note that the initial height would be the y component of the total distance traveled (d+x) so $h = (d+x)\sin\theta$
$U_f - U_i + U_{sp(F)} - U_{sp(i)}$
$0 - mgh + \frac{1}{2} k\Delta x^2 - 0$ $\frac{1}{2} kx^2 - mg(d+x)\sin\theta$

c. To determine the coefficient of kinetic friction, plug the term above back into the work-energy relationship, sub in –Work of friction as the work term and then solve for μ_k
$W_{NC} = \frac{1}{2} kx^2 - mg(d+x)\sin\theta$ $-f_k(d+x) = \frac{1}{2} kx^2 - mg(d+x)\sin\theta$

$$-\mu_k\, mg \cos\theta\,(d+x) = \frac{1}{2} kx^2 - mg(d+x)\sin\theta$$

$$\mu_k = [mg(d+x)\sin\theta - \frac{1}{2} kx^2] / [mg\,(d+x)\cos\theta]$$

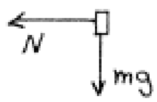
<u>1989M1</u>

a. Apply energy conservation from point A to point C setting point C as h=0 location (note: to find h as shown in the diagram, we will have to add in the initial 0.5m below h=0 location)

$U_A = K_C$ $\quad mgh_a = ½\, m\, v_c^2$ $\qquad (0.1)(9.8)(h_a) = ½\,(0.1)(4)^2$ $\qquad h_a = 0.816\text{m}$

$h = h_a + 0.5\text{ m} = 1.32\text{ m}$

b.



c. Since the height at B and the height at C are the same, they would have to have the same velocities $v_b = 4$ m/s

$F_{net(c)} = mv^2 / r$ $\qquad F_n = (0.1)(4)^2/(0.5) = 3.2$ N

d. Using projectile methods … $V_{iy} = 4\sin 30 = 2$ m/s $\qquad$ Then $\quad v_{fy}^2 = v_{iy}^2 + 2\, a\, d_y$

$(0) = (2)^2 + 2(–9.8)(d_y)$ $\qquad d_y = 0.2$

$h_{max} = d_y + \text{initial height} \qquad = 0.7$ m

Alternatively you can do energy conservation setting h=0 at point C. Then $K_c = U_{top} + K_{top}$ keeping in mind that at the top the block has a kinetic energy related to its velocity there which is the same as v_x at point C.

e. Since the block will have the same total energy at point C as before but it will lose energy on the track the new initial height h is larger than before. To find the loss of energy on the track, you can simply subtract the initial energies in each case. $\qquad U_{new} – U_{old} = mgh_{net} - mgh_{old}$

$(0.1)(9.8)(2\text{-}1.32) = 0.67$ J lost.

2003M1

a. $v = dx/dt = 1.5t^2 + 2$
   $v_0 = 2$ m/s

b. i. $K = ½ mv^2 = 50(1.5t^2 + 2)^2$
   ii. $F_{net} = ma = m(dv/dt) = m(3\ t) = 300\ t$
   iii. $P = Fv = (300\ t)(1.5t^2 + 2) = 450t^3 + 600t$ OR $P = dK/dt$

c. $W = \Delta K$; $v(2) = 8$ m/s, $v(0) = 2$ m/s; $W = ½\ m(8\ m/s)^2 – ½\ m(2\ m/s)^2 = 3000$ J
   Alternately, $W = \int P dt$

d. Greater, the student had to perform work against friction